

# A CAUSAL DECOUPLING APPROACH TO EFFICIENT PLANNING FOR LOGISTIC PROBLEMS WITH STATEFUL STOCHASTIC DEMAND



Grants IIS-1849303, IIS-1849249, and IIS-1849291.

**DEMAND** Diptanil Chaudhuri and Dylan Shell  
Texas A&M University

## Overview

- Inclusion of sophisticated, time-correlated stochastic models of demand is the key to improving flexibility in modern logistic planning.
- Dynamic models of demand, unfortunately, can increase the space in which planning occurs when treated via the Markov Decision Process formulation.
- In this paper, we subdue the growth in complexity via decoupling.

## Problem statement

**Given:** A set of commodities, a logistic network, set of demand models, a vertex mapping function, and an initial storage.

**Output:** A policy of valid actions that minimizes the expected time for all commodities to be consumed.

## Solution to logistic problem via decoupling

1 When there are multiple sites and different influence on demand at these sites, they can be factored by splitting into separate site-specific models.

2 Though one may not know future demand, one can usually determine current demand through suitable instrumentation (say, via market analysis).

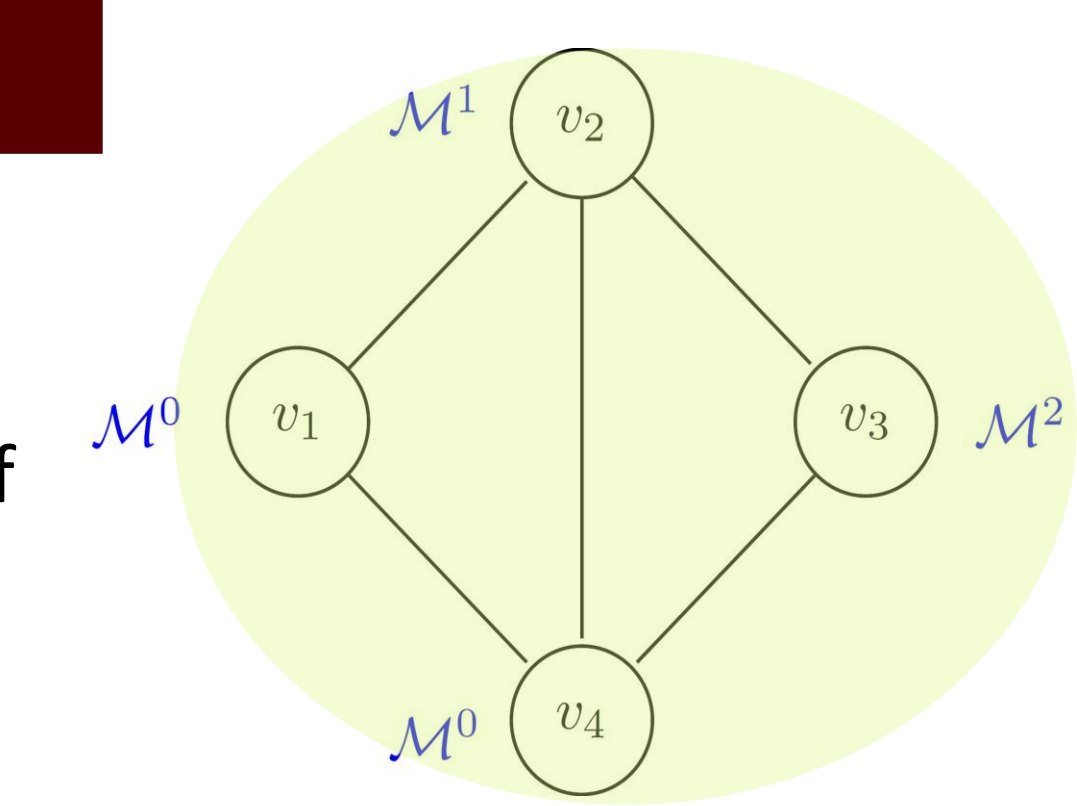
3 State transitions within demand model reflect aspect of the stochastic process which describe uncertainty.

Decoupled approach to approximate solutions

Retaining some amount of temporal structure provides useful compromise between quality of solution obtained and computation required.

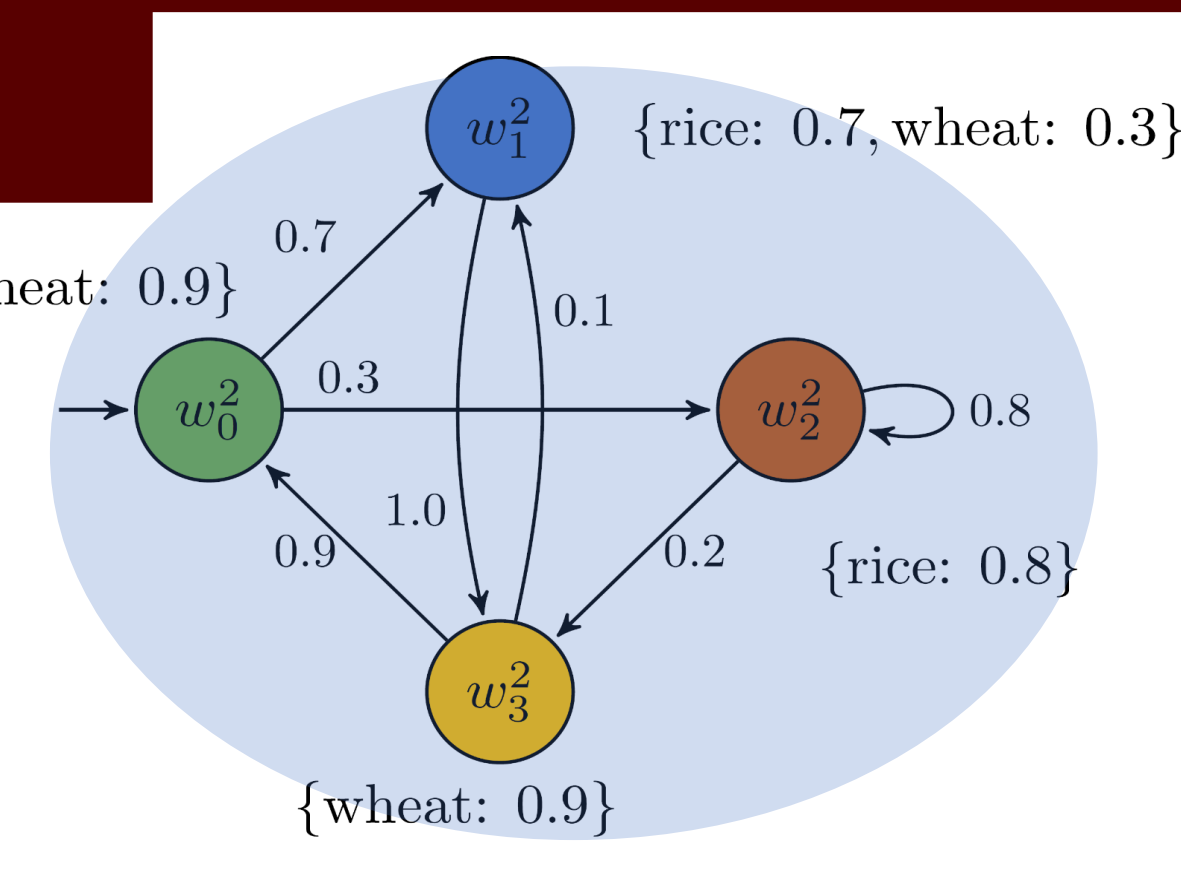
## Logistic network

A logistic network is a 4-tuple  $L = (V, E, S, U)$  where: (1)  $V$  is the non-empty set of vertices; (2)  $E \subseteq V \times V$  be the symmetric directed edges of the network; (3)  $S$  is the vertex storage capacity function; and (4)  $U$  is the edge bandwidth function.



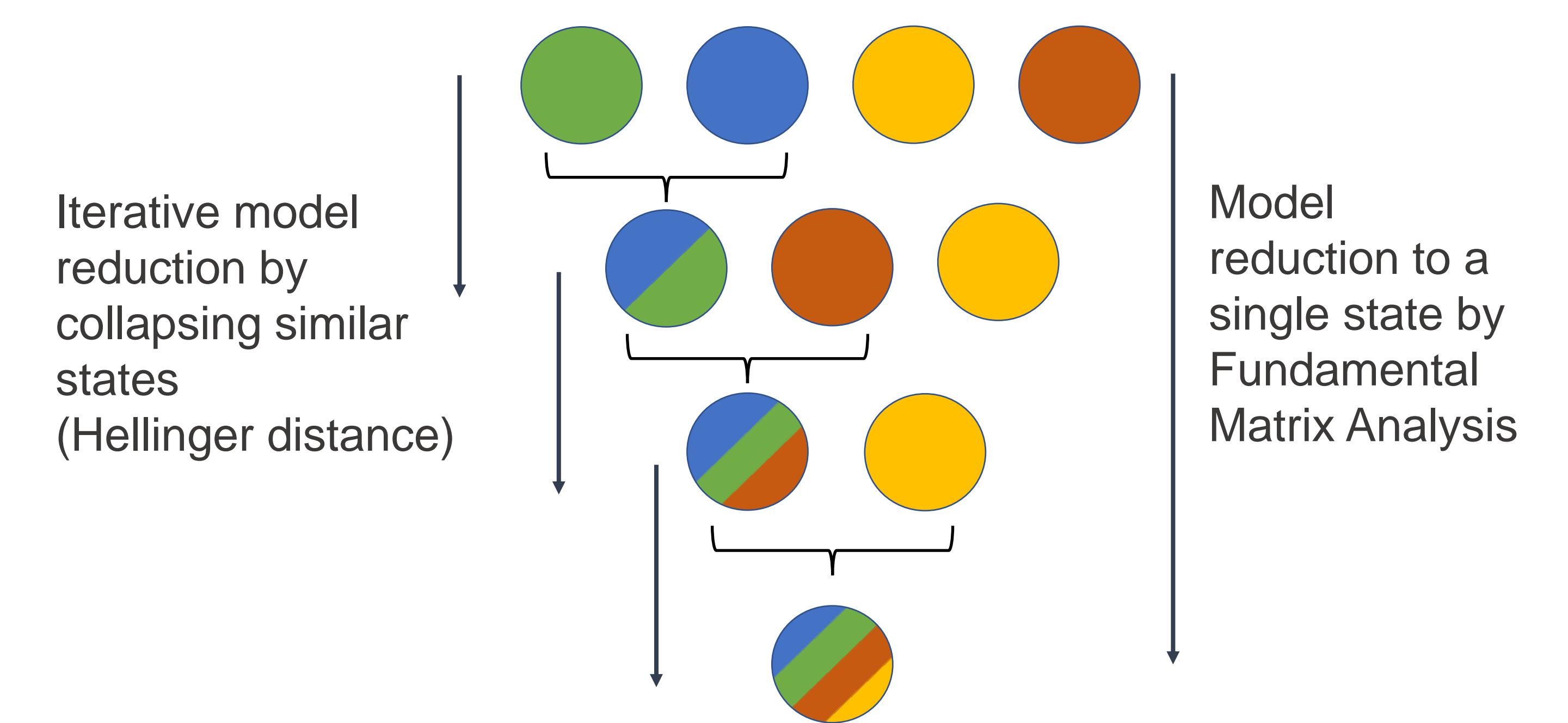
## Demand model

A demand model is a 5-tuple  $M = (W, C, w_0, \tau, \delta)$  where (1)  $W$  is the non-empty finite state space with  $w_0$  being the initial state; (2)  $C$  is the commodity set; (3)  $\tau: W \times W \rightarrow [0, 1]$  is the transition probability function; and (4)  $\delta: W \times C \rightarrow [0, 1]$  is the demand function.



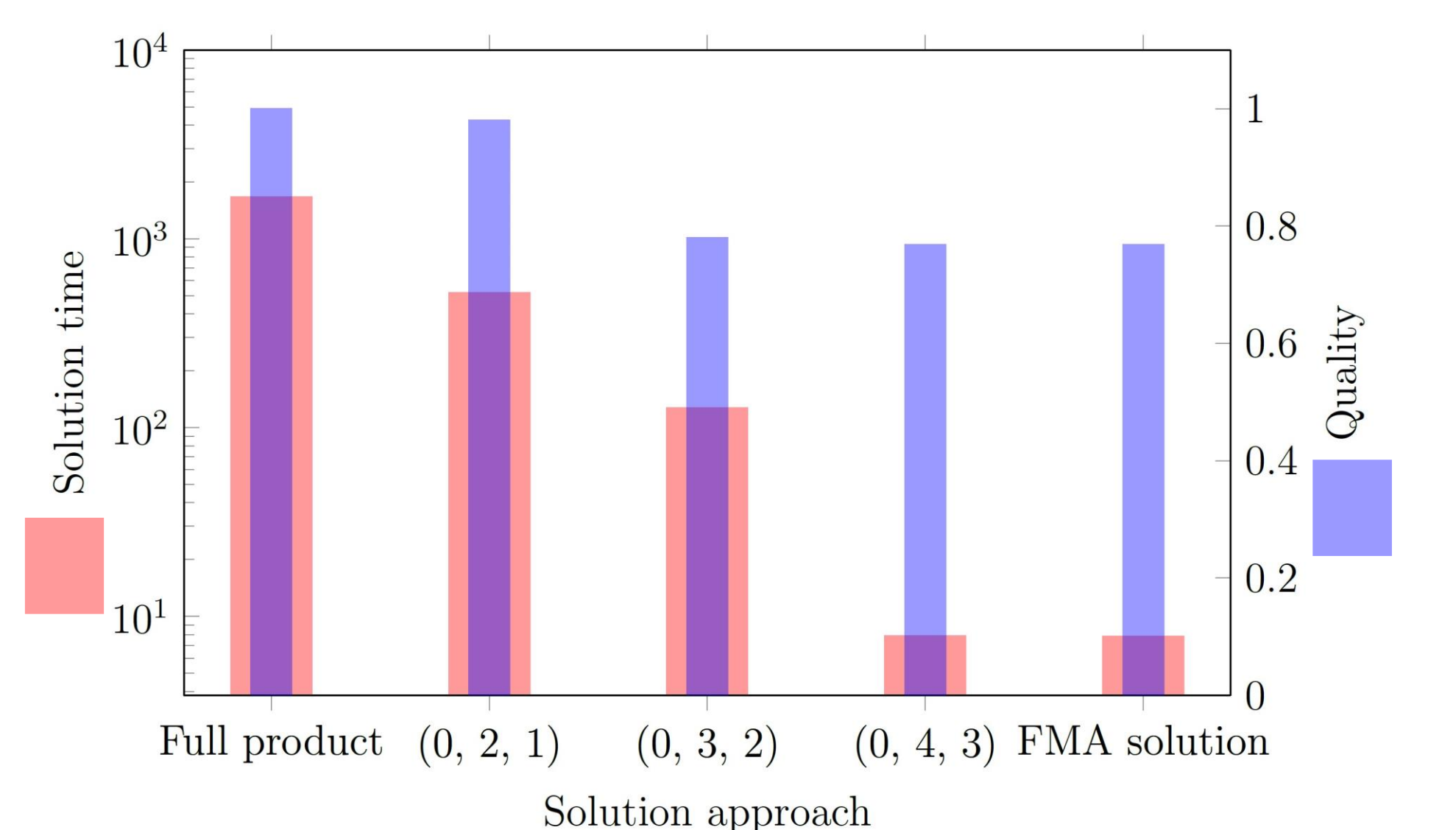
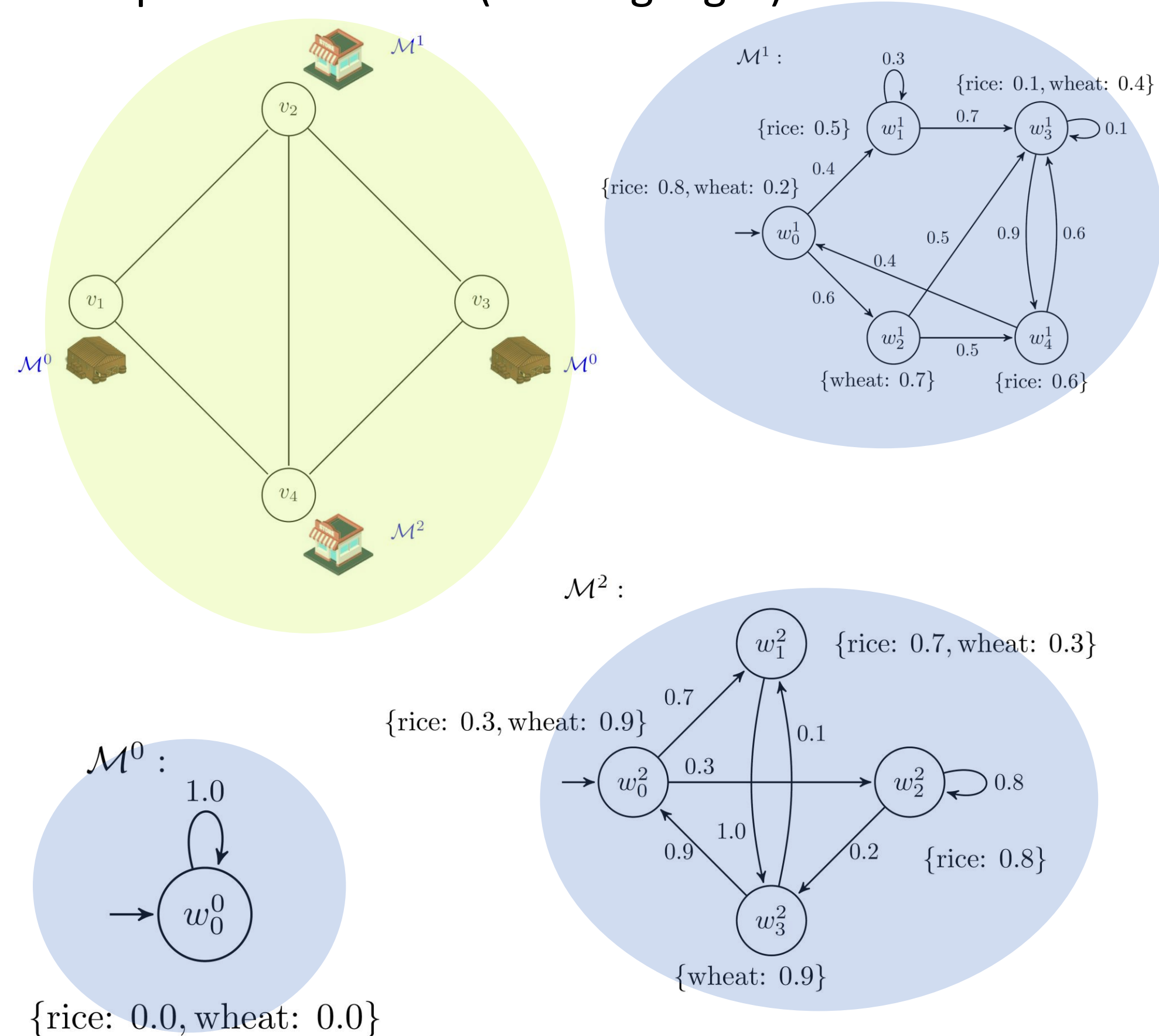
## Model reduction – reducing temporal correlation

The number states in each demand model increases the planning problem multiplicatively. The non-causality of the demand model provides opportunity to simplify and analyze them independently beforehand.



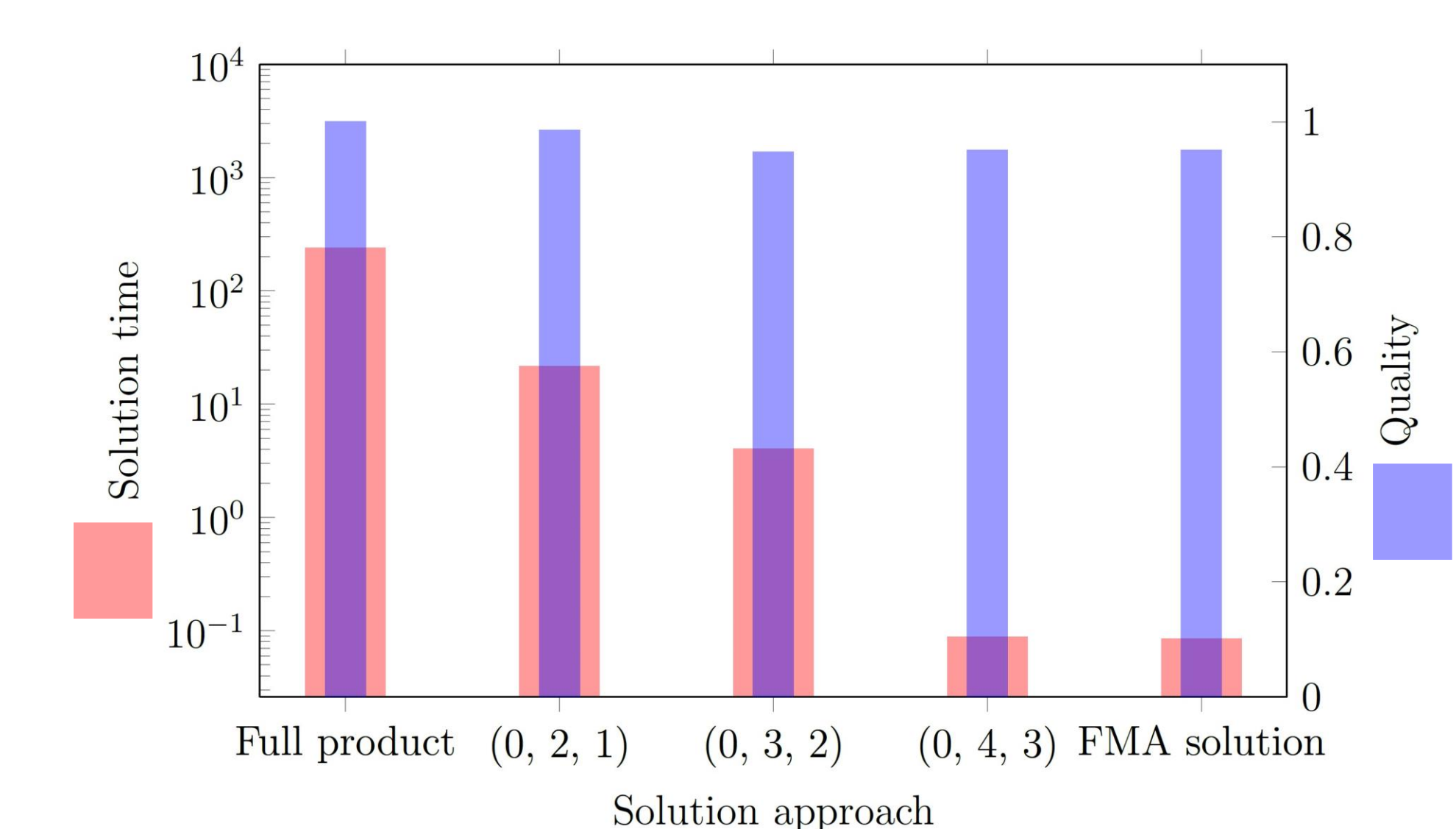
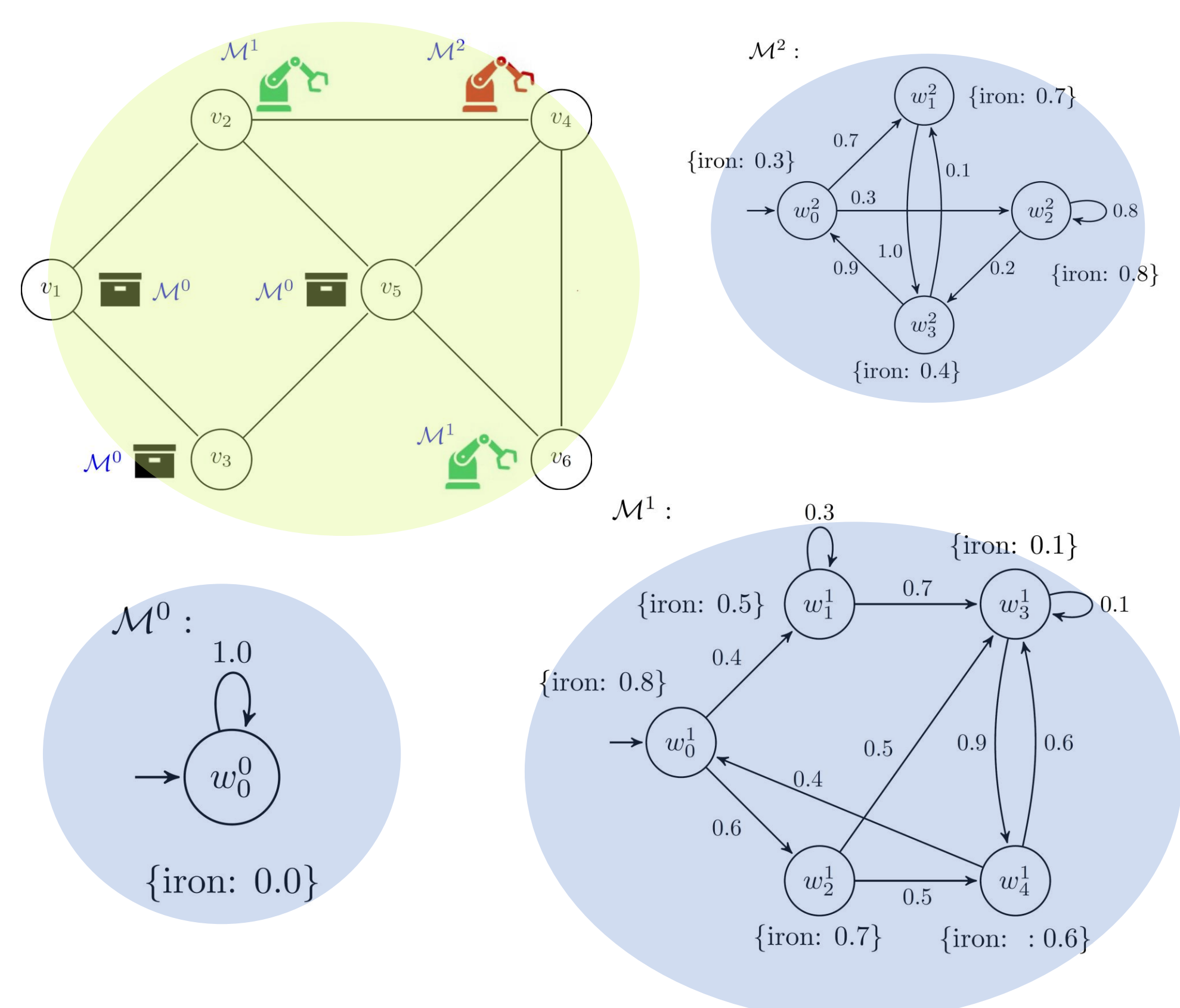
## Routing grain: rice and wheat

An autonomous operation agent is responsible for routing multiple commodities (rice and wheat) within a logistic network (green highlight) having site-specific demands (blue highlight).



## Lean manufacturing

An autonomous agent in a lean manufacturing factory floor (green highlight) producing nails and screws must transport raw materials in from of iron bars. Assume that the demand for nails and screws is directly reflected on the demand for iron bars (blue highlight).



## FMA reduction

FMA approach uses matrix analysis on each demand model to collapse all the states into a single state. This collapse of states destroys the temporal structure of the original demand models and reduces the dynamics of consumer demand for every commodity into a Bernoulli random variable.

## Collapsing state pairs

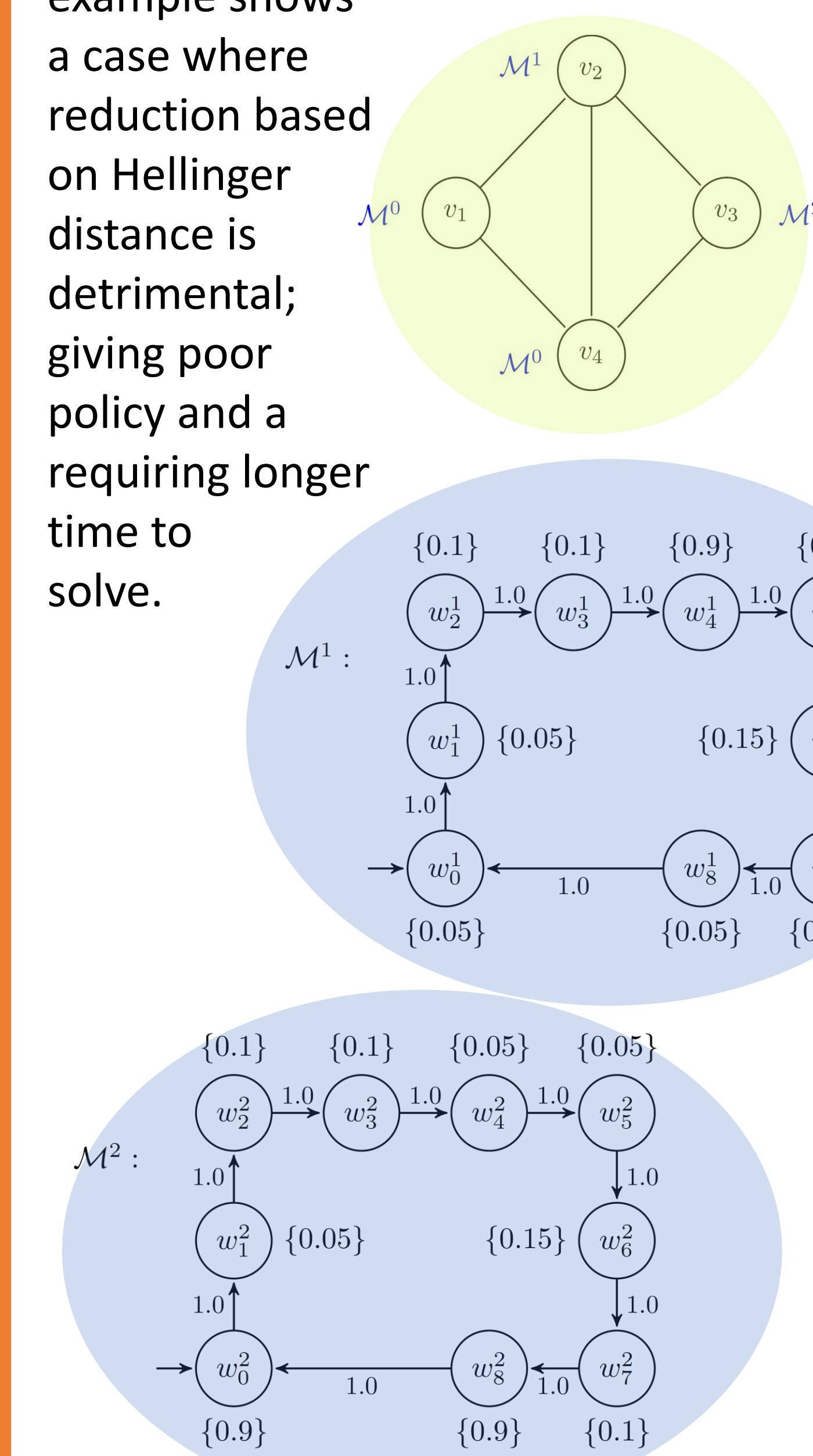
The Hellinger distance approach provides a method to collapse two similar states into one. Therefore, it gives a spectrum of in-between reductions as it can be applied to the original demand function iteratively.

- Each state of the demand model is associated with two distributions: (a) distribution over the states of the demand model, given by the transition function  $\tau(w, \cdot)$ ; and (b) joint probability distribution over every commodity derived from the demand function  $\delta(w, \cdot)$ .
- To quantify the similarity between two states, we need to quantify the similarity between their two distributions.
- We introduce a modified formulation of the Hellinger distance with parameter  $\alpha$  that assigns preference of one distribution over the other.

## Drawback of reduction via FMA and Hellinger distance.

Model reduction approaches presented here are few of many approaches that can be used to pre-analyze the demand model in order to generate efficient approximate solutions. They are myopic and other approaches may provide better efficiency.

This constructed example shows a case where reduction based on Hellinger distance is detrimental; giving poor policy and a requiring longer time to solve.



Manual reduction (right) performs better than reduction via Hellinger distance or FMA reduction.

